Midterm 390
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Test

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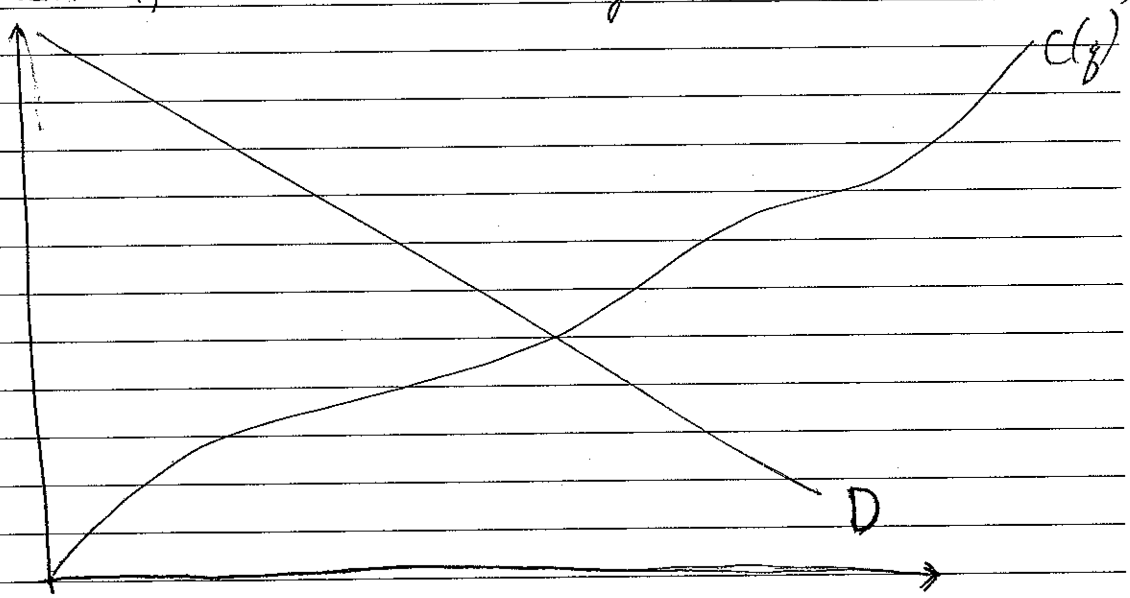
Book No. 1 of 1 Books

Subject ECON 390 Section /

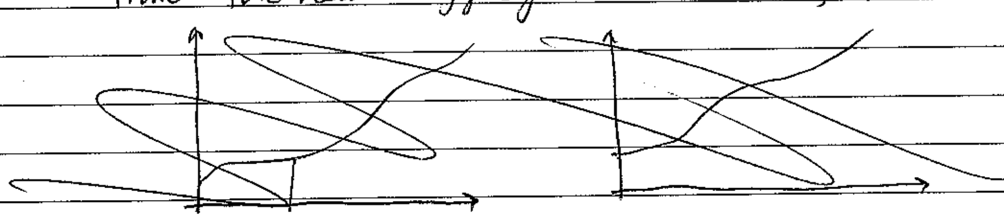
②. Consider a model wherein each firm has 1 tonne of oil, so that no firm has market power and the market is competitive. These firms vary in the cost of extracting their tonne of oil, for example, some deposits are deeper than others or require more refining.

We could arrange the ~~firms~~ tonnes of oil in order of ~~easiest~~ cheapest to most expensive to extract, and build an industry cost of extraction curve, where every firm represents one point on the curve

Let D be the aggregate demand facing the industry.

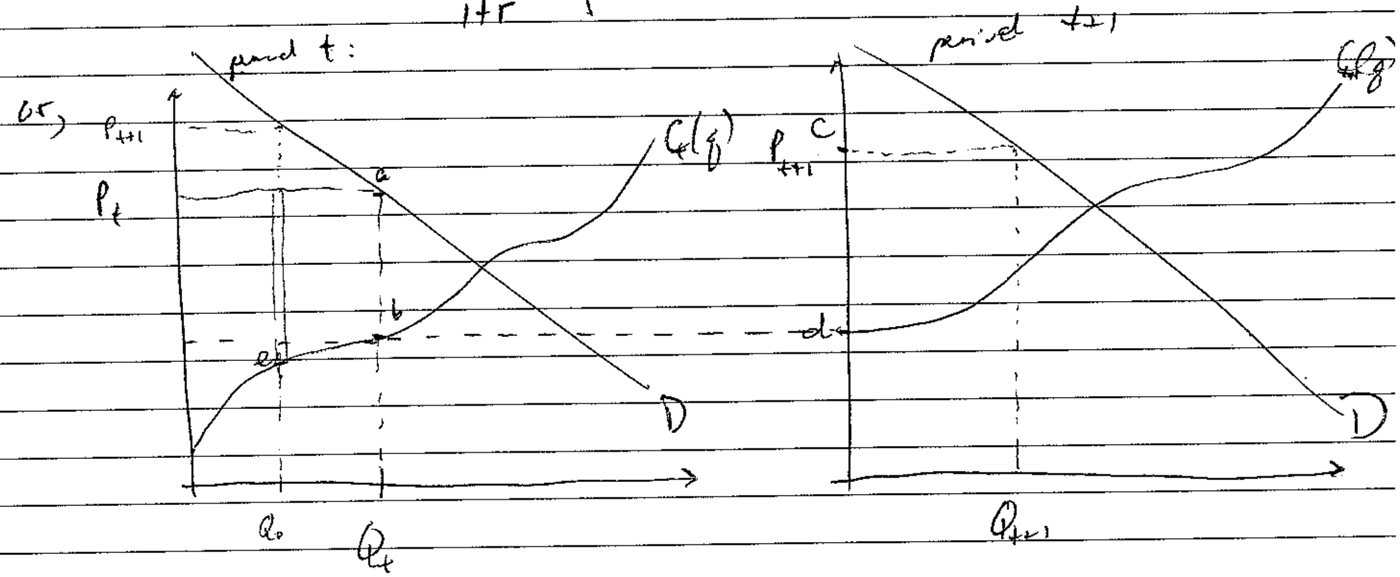


~~Equilibrium~~ In each period, some amount of oil & associated firms Q_t is extracted and their oil sold, in the period $t+1$, these firms are absent from the new aggregate cost curve,



Competitive equilibrium requires that the marginal firm (f_i) in period t is indifferent to extracting in t or in $t+1$, so the present value of both options are equal, i.e.

$$P_t - C(f_i) = \frac{1}{1+r}\left(P_{t+1} - C(f_i)\right),$$



So the distance ab is equal to the distance cd times $(1+r)$. ✓ The marginal firm, producing at ab, is indifferent in this case (between periods), while an intra-marginal firm, say the one associated with Q_0, prefers to produce in period t due to his lower costs. If he delayed he would recieve $P_{t+1} - e$ in profit in period 2, however, $P_t - e > \frac{1}{1+r}(P_{t+1} - e)$, since ~~the margin~~ it is the "margins" which are growing at $r\%$. ✓

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(4) Solow's model of inter-generational sustainability follows from a model where there is some production function $F_t(R_t, K_t)$, where R is the amount of some resource (e.g. oil) consumed in t, and K_t is capital stock.

In this case,

$$C_t = F_t(R_t K_t) - \frac{dK_t}{dt}$$, since output is either consumed or invested.

Then $$\frac{dC_t}{dt} = \frac{dR}{dt} \cdot F_R + \frac{dK}{dt} F_K - \frac{d^2 K}{dt^2}.$$

Consider $\frac{dK}{dt} = R \cdot F_R$ = current resource rent.

$$\frac{d^2K}{dt^2} = \frac{dR}{dt} \cdot f_R + \frac{dF_R}{dt} \cdot R$$

So $$\frac{dC_t}{dt} = \cancel{\frac{dR}{dt} \cdot F_R} + \frac{dK}{dt} F_K - \cancel{\frac{dR}{dt} \cdot F_R} - \frac{dF_R}{dt} \cdot R$$

$$= R \cdot F_R \cdot F_K - R \cdot \frac{dF_R}{dt}$$

$$= R\left(F_R \cdot F_K - \frac{dF_R}{dt}\right)$$

but from Hotelling extraction, we know that

$F_R \cdot F_K = \frac{dF_R}{dt}$, since $F_K = \frac{\frac{dF_R}{dt}}{F_R}$

interest rate

% growth in price of oil.

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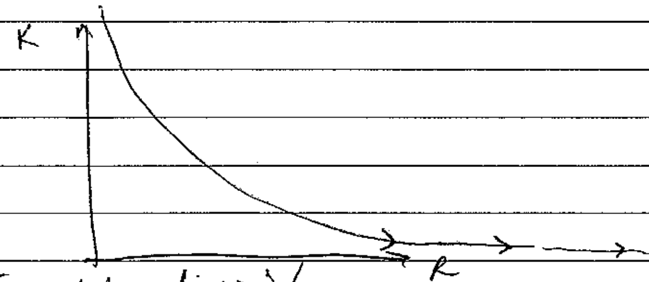
This work is based on 2 assumptions:

1) Resources are infinitely divisible, so that any arbitrarily small amount can be used at a time.

The resource use in this model essentially moves the economy along a ~~R~~ Cobb-Douglas capital-~~oil~~ isoquant,

so as $t \to \infty$, $R \to 0$.



✓ However, resources are not infinitely divisible.

For example one cannot have less than an atom of oil.

✓ 2) Substitutability of oil and capital.

The use of a Cobb-Douglas production function implies that for any given output y_0, and an arbitrarily small non-zero amount of oil R_0, there exists a high enough level of capital K_0 so that y_0 can be produced. This is not feasible, for example, Given only one drop of oil no amount of capital, i.e. no ~~[illegible]~~ large enough &/or efficient enough furnace, exists to heat a house with only 1 drop.

(10)

(1) See question (4) for development of Solow model.

In the Kuwait sustainable investment model, consider an oil exporter with capital fund F_t. In every period, their profits (from sale of oil and return on F_t) are divided between consumption and investment in the manner:

$$\pi(q_t^*) + r \cdot F_t = C_t + I_t, \text{ or}$$

$$C_t = r \cdot F_t + \pi(q_t^*) - I_t$$

Notice that $\frac{dF_t}{dt} = I_t$.

Consider I_t = current rent = $q_t^* \cdot \frac{d\pi(q_t^*)}{dq_t^*}$

Then $$\frac{dC_t}{dt} = r \cdot \frac{dF_t}{dt} + \frac{d}{dt}\pi(q_t) - \frac{dq_t}{dt} \cdot \frac{d\pi(q_t)}{dt} - \frac{d}{dt}\left(\frac{d\pi(q_t)}{dq_t}\right) \cdot q_t$$

$$= r \cdot q_t \cdot \frac{d\pi(q_t)}{dq_t} + \frac{d}{dt}(\pi(q_t)) - \frac{dq_t}{dt} \cdot \frac{d\pi(q_t)}{dt} - q_t \cdot \frac{d}{dt}\left(\frac{d\pi(q_t)}{dq_t}\right)$$

since $\frac{d}{dt}(\pi(q_t)) = \frac{d\pi(q_t)}{dq_t} \cdot \frac{dq_t}{dt}$, (by chain rule)

$$\frac{dC_t}{dt} = r \cdot q_t \cdot \frac{d\pi(q_t)}{dq_t} + \cancel{\frac{dq_t}{dt} \cdot \frac{d\pi(q_t)}{dq_t}} - \cancel{\frac{dq_t}{dt} \cdot \frac{d\pi(q_t)}{dt}} - q_t \cdot \frac{d}{dt}\left(\frac{d\pi(q_t)}{dq_t}\right)$$

Now, if oil is extracted according to Hotelling model, then $\frac{d}{dt}\left(\frac{d\pi(q_t)}{dq_t}\right)$

$= \frac{d}{dt}\left(P - MC(q_t)\right) = r \cdot \left(P - MC(q_t)\right) = r \cdot \frac{d\pi(q_t)}{dt}$, i.e. rents grow at r percent.

so $\frac{dC_t}{dt} = r \cdot q_t \cdot \frac{d\pi(q_t)}{dt} \mp r \cdot q_t \frac{d\pi(q_t)}{dt}$

$= 0.$

Both The Solow and Kuwait models are very similiar. They both rest on production functions which are functions of accumulated capital and oil extracted in period t.

$F(K_t, R_t)$ in Solow, $r \cdot F_t$ in Kuwait → through $\pi(q_t)$ in Kuwait, through $F(K_t, R_t)$ in Solow.

Both assume that all production is either consumed or invested, and both rely on the assumption that oil is being extracted in the "correct" manner, i.e. so that present value of lifetime profits are maximised and $(P - MC(q_t))$ grows at r percent.

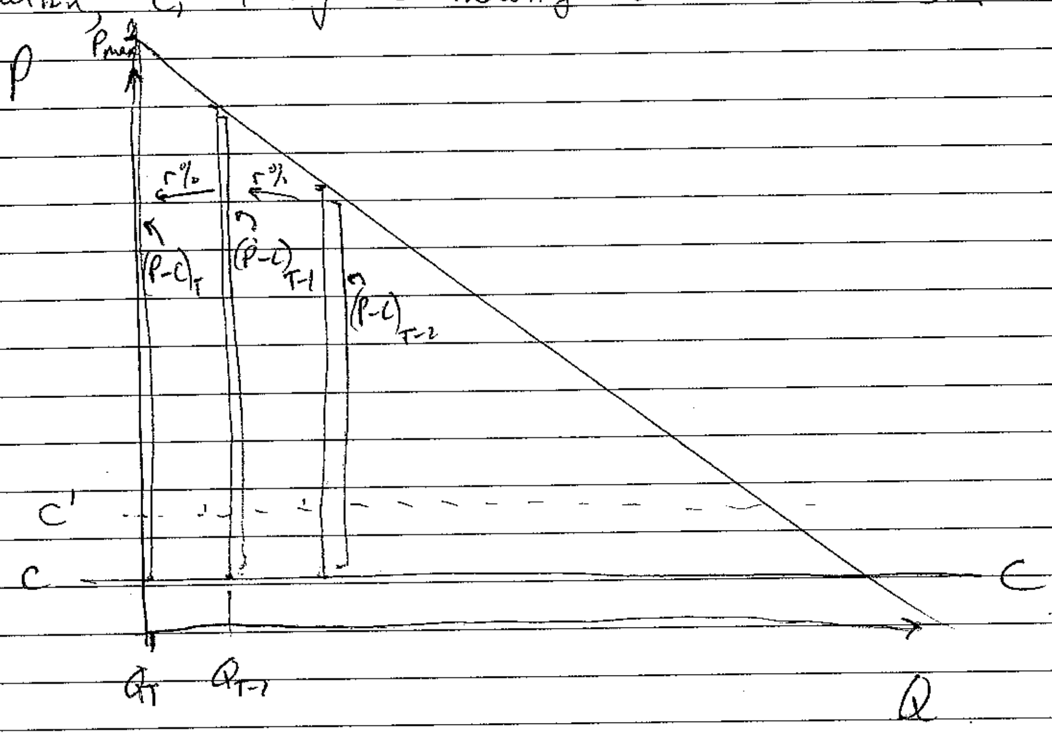
Both find that sustainable investment occurs when current rents, i.e. $q_t \cdot \frac{\partial\pi(q_t)}{\partial q_t}$ are invested.

The 2 principles are:

1) extract resources in competitive Hotelling manner, i.e. so that $\frac{\partial\pi(q_t)}{\partial q_t}$ rises at r%.

2) Choose a level of investment so that consumption is constant over time, ~~so~~. ~~now~~ this occurs when current resource rents are invested. Then the increase in investment return over time offsets the decrease in oil profits.

(5) Consider a competitive industry with constant marginal extraction cost of extraction, C, facing an industry demand curve D.



The competitive industry will follow the extraction path that maximises total surplus in the industry (assuming perfect information, no market power, etc). From this, we can determine the extraction path.

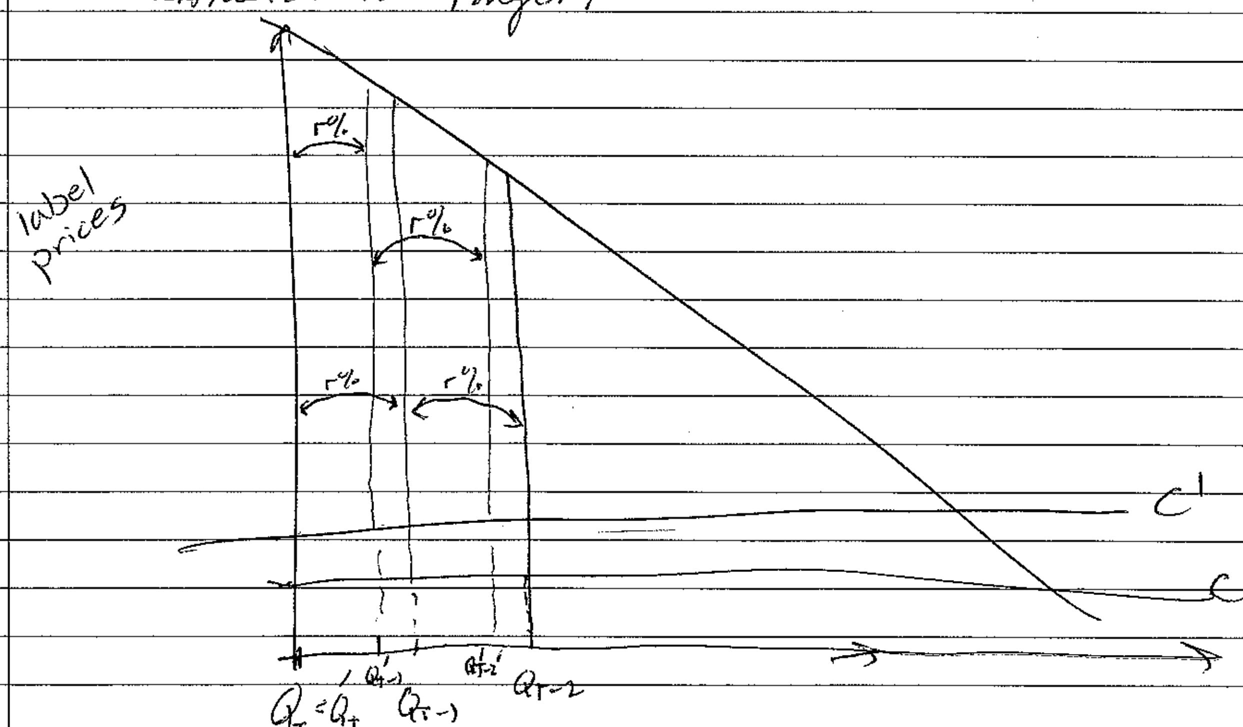
In the final period, T, we need to satisfy that the marginal ~~cost~~ benefit of extracting ~~....~~ Q_T ~~now~~ is equal to the average benefit of extracting Q_T, this is satisfied only if Q_T is infintesmaly small.

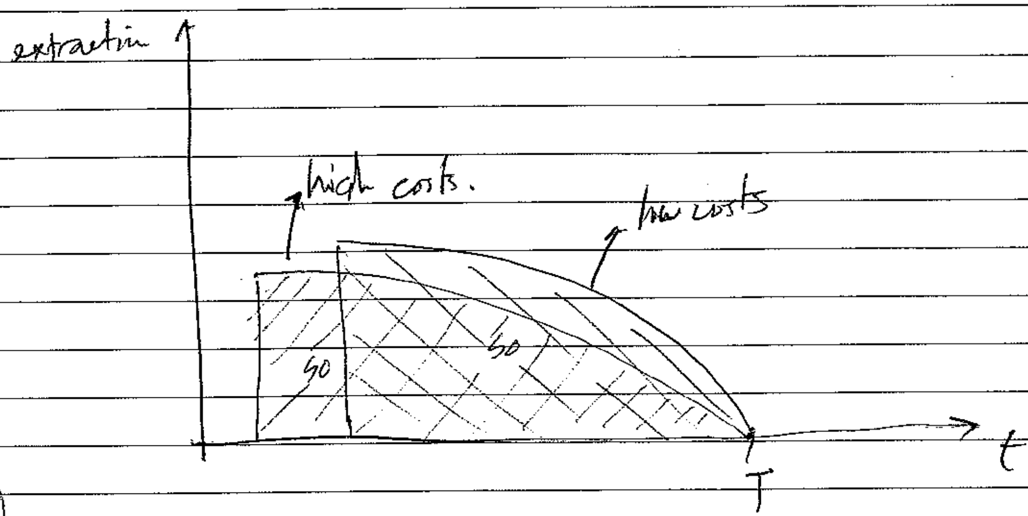
Working backwards from this we know that marginal net benefit, which is Price minus costs C at Q_t, must shrink by r% every period. See graph above.

If C rises to C' and we apply the same logic, we can determine the new extraction quantities Q_T', Q_{T-1}', etc.

Q_T' must still satisfy marginal benefit = avg benefit, so Q_T' also infinitesimally small. Then Q_{T-1}' satisfies $(P-C')_{T-1}$ is r% less than $(P-C')_T$, and so on.

Since $P_{max}-C' < P_{max}-C$, then $P_T > P_T'$ for r% growth to occur, and so on in periods $T-2$, $T-3$, etc. In other words, since costs are higher, prices must fall more slowly in order for r% growth in $(P-C)$ to occur. This implies $Q_{T-1}' < Q_{T-1}$, $Q_{T-2}' < Q_{T-2}$, etc. Since under higher costs less is extracted in every period, it is extracted for longer.





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(6) In the open-loop case, each firm chooses an extraction path, treating the opponent's as fixed, in order to maximise net present value of profits. Once they have each created a best-response extraction path, with full information, they are committed and the "game" is played out. They are naive in the sense that they do not re-evaluate their extraction path strategy at each period and adjust accordingly, but rather follow the path chosen originally blindly. closed A more sophisticated model would have each player creating a best-response path in every period, and playing at sequentially in this manner. For illustration, consider a monopolist against a competitive fringe. The monopolist may do best to sell some stock right from the beginning, pushing the fringe to exhaust its supply sooner and allowing the monopolist to extract monopoly profits on its remaining stock sooner. However, once the fringe has committed to exhausting their stock sooner, since the monopolist is also selling right from the start, then the monopolist no longer has incentive to sell in the initial period, and will default, hoarding stock until the fringe is exhausted. A naive ~~strategy~~ model ~~will~~ will not incorporate the monopolists incentive to defect once the fringe is

3 $\cancel{V_{stock} = P_t \cdot V_t}$

committed, since the monopolist will not re-evaluate and adjust his strategy once the clock has started, so to speak.

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3. Miller and Upton conducted an empirical test on Hotelling's simple r% calculation based on the competitive industry. They consolidated Hotelling's equation into fit into their analysis of 69 oil firm data sample.

According to Hotelling, a value of firm is the sum of PV of future profits

$$V(S_t) = \pi(q_t) + \pi(q_{t+1})\left(\frac{1}{1+r}\right) + \pi(q_{t+2})\left(\frac{1}{1+r}\right)^2 \dots + \pi(q_{T-t})\left(\frac{1}{1+r}\right)^{T-t}$$

If Hotelling's idea is true, then

$$\pi(q_{t+1})\left(\frac{1}{\cancel{1+r}}\right)\cancel{(1+r)} = \pi(q_t)$$

$$\pi(q_{t+2})\left(\frac{1}{\cancel{1+r}}\right)^2\cancel{(1+r)}^2 = \pi(q_{t+1})$$

$$\pi(q_{t+3})\left(\frac{1}{\cancel{1+r}}\right)^3\cancel{(1+r)}^3 = \pi(q_{t+2})$$

The r% discount rate doesn't matter in the heterogeneous world of oil

then $V(S_t) = \underbrace{(pq - c(q))}_{\text{profit}} \times \underbrace{(q_0 + q_1 + \dots q_t)}_{\text{total quantity of oil stock}}$

$= (pq - c(q))\, S_0$

Applying Hotelling's theory to their regression.

Miller+Upton have the stock value of firm $= V_t$, S_t = oil supply

$$\frac{V_t}{S_t} = a + b(\underbrace{p_t - c_t}_{\text{marginal profit}})$$

If Hotelling is true then $a = 0$, $b = 1$, oil firms are homogeneous.

They got a result: $a \lesssim 0$, $b \approx 0.92$ which was very close to Hotelling's simple r% model.

However, their analysis was inaccurate and flawed. This is because the 69 firms from the NYSE each faces a different marginal cost with a heterogeneous quality of oil. Hence, the oil price is different for each.

$c^i \neq c^j$ $\quad p^i \neq p^j$

Therefore, by consolidating the Hotelling's r% equation into their regression analysis, they're applying the wrong heterogeneous data into Hotelling's simple homogeneous r% rule